Tokenomics and Platform Finance

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Macro Finance Society
Digital Platforms and Tokens

- The rise of digital platforms
  - Payment innovation is important, e.g., escrow account on eBay and Alibaba

- Tokens: users’ means of payments on platform
  - Blockchain: preventing double spending, facilitating smart contracts

- Tokens: platforms’ financing instruments
  - Token offerings $ 21 billion in 2018; US VC $ 131 billion
  - Tokens used to gather resources (e.g., engineers, consultants, investors)
  - Tokens enter into circulation gradually (protocol and vesting)

- Tokens: rewards for the founding entrepreneurs
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Outline

- Introduction
- Model and Solution
  - Franchise Value as Discipline – Durable-Goods Monopoly
  - Token Overhang – Corporate Finance
  - The Value of Commitment – Time Inconsistency
- Conclusion
A platform supports a unique set of transactions

User $i$ settles transactions in tokens, deriving *convenience yield* from token value

- Efficient payment, smart contracting...
A platform supports a unique set of transactions

- Productivity: $A_t$

User $i$ settles transactions in tokens, deriving convenience yield from token value $x_{i,t} = P_t k_{i,t}$

- Convenience yield: $x_{i,t}^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha dt$
  - Token price: $P_t$
  - Token units: $k_{i,t}$
  - Number of users: $N_t$
  - User heterogeneity: $u_i \sim G_t(u)$
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• Token price appreciation $k_{i,t} E_t[dp_t]$

Token price dynamics in equilibrium

\[
\frac{dp_t}{P_t} = \mu_t^p dt + \sigma_t^p dZ_t
\]
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  - User heterogeneity: \( u_i \sim G_t(u) \)
- Token price appreciation \( k_{i,t} E_t [dP_t] \)
- Participation cost \( \phi dt, \text{ if } k_{i,t} > 0 \)

\[
N_t = 1 - G_t(u_t)
\]
Objective

\[ \int_{t=0}^{+\infty} e^{-rt} \left[ \max\{0, \text{convenience} + \text{net token return} - \text{participation cost} \} \right] dt \]
Token Demand

\[ k_{i,t} = \frac{F(E_t[dP_t], A_t)}{P_t} u_i \]

\[ \frac{\partial F}{\partial E_t[dP_t]} > 0 \]

\[ \frac{\partial F}{\partial A_t} > 0 \]
Token Market Clearing

\[ M_t = \int_{u=u_t} \frac{F \left( E_t[dP_t], A_t \right)}{P_t} u dG_t(u) \]
Token Market Clearing

\[ M_t = \frac{F(E_t[dP_t], A_t)}{P_t} \int_{u=u_t}^{u} udG_t(u) \]

- \( P_t \) decreases in supply \( M_t \), increases in \( A_t \)
- 1\textsuperscript{st}, 2\textsuperscript{nd} order derivatives in \( E_t[dP_t] \) by Itô's lemma
  \( \Rightarrow \) Differential equation for \( P_t = P(M_t, A_t) \)
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How do the state variables $A_t$ and $M_t$ evolve?

Token Market Clearing

$$M_t = \frac{F(E_t[dP_t], A_t)}{P_t} \int_{u=u_t} udG_t(u)$$

- $P_t$ decreases in supply $M_t$, increases in $A_t$
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- Productivity: \( \frac{dA_t}{A_t} = L_t dH_t \)
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- **Contributor** resource: *endogenous* \( L_t \)
- **Entrepreneur** contribution: \( dH_t = \mu^H dt + \sigma^H dZ_t \)
A platform supports a unique set of transactions

- Productivity: \[ \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t) \]
- Platform investment: endogenous \( L_t \)
A platform supports a unique set of transactions

- Productivity:
  \[ \frac{dA_t}{A_t} = L_t(\mu^H dt + \sigma^H dZ_t) \]

- Platform investment:
  \[ \text{endogenous } L_t \]

Tokens paid

\[ \frac{F(L_t, A_t)dt}{P_t} \]

Token Supply

\[ dM_t = \frac{F(L_t, A_t)dt}{P_t} \]
A platform supports a unique set of transactions

- Productivity: \[ \frac{dA_t}{A_t} = L_t(\mu^H dt + \sigma^H dZ_t) \]
- Platform investment: endogenous \( L_t \)
- Tokens paid to owner (cumulative): \( D_t \)

\[
\text{Token Supply} \quad dM_t = \frac{F(L_t, A_t)dt}{P_t}
\]
A platform supports a unique set of transactions

- Productivity:
  \[
  \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t)
  \]
- Platform investment: endogenous \( L_t \)
- Tokens paid to owner: \( dD_t > 0 \)
- Tokens burnt by owner: \( dD_t < 0 \)

Token Supply

\[
dM_t = \frac{F(L_t, A_t) dt}{P_t} + dD_t
\]
A **platform** supports a unique set of transactions

- **Productivity:**
  \[ \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t) \]

- **Platform investment:** *endogenous* \( L_t \)

- **Tokens paid to owner:**
  \[ dD_t > 0 \]

- **Tokens burnt by owner:**
  \[ dD_t < 0 \]

**Token Supply**

\[ dM_t = \frac{F(L_t, A_t)dt}{P_t} + dD_t \]
\[
max_{\{L_t, dD_t\}} \int_{t=0}^{+\infty} e^{-rt} P_t dD_t [I_{\{dD_t \geq 0\}} + (1 + \chi) I_{\{dD_t < 0\}}] dt
\]

- Token buy-back financing cost: \( \chi \)
\[
\max_{\{L_t, dD_t\}} \int_{t=0}^{+\infty} e^{-rt} P_t dD_t \left[ I_{\{dD_t \geq 0\}} + (1 + \chi) I_{\{dD_t < 0\}} \right] dt
\]

- \( V_t = V(M_t, A_t) \), \( \frac{\partial V}{\partial M} < 0 \), \( \frac{\partial V}{\partial A} > 0 \)
- HJB is differential equation for \( V(M_t, A_t) \)

\[
dM_t = \frac{F(L_t, A_t) dt}{P_t} + dD_t \quad \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t)
\]
$$\max_{\{\mu_t,dD_t\}} \int_{t=0}^{+\infty} e^{-rt} P_t dD_t \left[ I_{\{dD_t \geq 0\}} + (1 + \chi) I_{\{dD_t < 0\}} \right] dt$$

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- \( P_t \) decreases in supply \( M_t \), increases in \( A_t \)

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\]
A platform supports a unique set of transactions

- **Productivity:**
  \[
  \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t)
  \]
- **Contributor resource:**
  \[
  \text{Payment} = \frac{L_t}{F(L_t, A_t)} dt
  \]
- **Tokens paid to owner:**
  \[dD_t > 0\]
- **Tokens burnt by owner:**
  \[dD_t < 0\]

**User** \(i\) settles transactions in tokens, deriving *convenience yield* from token value \(x_{i,t} = P_t k_{i,t}\)

- **Convenience yield:**
  \[
  x_{i,t}^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha dt
  \]
  - **Token price:**
    \[P_t\]
  - **Token units:**
    \[k_{i,t}\]
  - **Number of users:**
    \[N_t\]
  - **User heterogeneity:**
    \[u_i \sim G(u)\]
- **Participation cost**
  \[\phi dt, \text{if } k_{i,t} > 0\]
- **Token price appreciation**
  \[E_t[dP_t]\]

**Objective**

\[
\int_{t=0}^{+\infty} e^{-rt} P_t dD_t \left[ I_{\{dD_t \geq 0\}} + (1 + \chi) I_{\{dD_t < 0\}} \right] dt
\]

- **Value**:
  \[V_t = V(M_t, A_t), \frac{\partial V}{\partial M} < 0, \frac{\partial V}{\partial A} > 0\]

**Token Supply**

\[dM_t = \frac{F(L_t, A_t) dt}{P_t} + dD_t\]

**Token Market Clearing**

\[M_t = \frac{F(E_t[dP_t], A_t)}{P_t} \int_{u=u_t} udG_t(u)\]

- **Token price**
  \[P_t\]
  - **Decreases in supply** \(M_t\), **increases in** \(A_t\)

**Token Price**

\[\frac{dP_t}{P_t} = \mu^P dt + \sigma^P dZ_t\]

**endogenous**
Transform the State Space

State space: \((M_t, A_t) \rightarrow (m_t, A_t)\), where \(m_t = \frac{M_t}{A_t}\)
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State space: \((M_t, A_t) \rightarrow (m_t, A_t)\), where \(m_t = \frac{M_t}{A_t}\)

\(V(M_t, A_t) = A_t v(m_t)\), and \(P(M_t, A_t) = P(m_t)\)

Solve ODEs of \(v(m_t)\) and \(P(m_t)\)
Transform the State Space

State space: \((M_t, A_t) \rightarrow (m_t, A_t)\), where \(m_t = \frac{M_t}{A_t}\)

\[ V(M_t, A_t) = A_t \nu(m_t) \text{, and } P(M_t, A_t) = P(m_t) \]

Solve ODEs of \(\nu(m_t)\) and \(P(m_t)\)

\[ \frac{\partial V}{\partial M_t} = \nu'(m_t) < 0 \quad P'(m_t) < 0 \]
Platform Owner Value Function

$v(m_t)$ vs. Token Supply / Platform Productivity $m_t$
Outline

- Introduction
- Model and Solution
- Franchise Value as Discipline
- Token Overhang
- The Value of Commitment
- Conclusion
Optimal Platform Payout and Buy-back (burn) $dD_t$
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\[ \frac{m}{m_t} \]

\[ dD_t < 0 \]

\[ -\frac{\partial V}{\partial M_t} = -\nu'(m_t) = P_t(1 + \chi) \]
Optimal Platform Payout and Buy-back (burn) $dD_t$

\[
\begin{align*}
\frac{m}{m} & \quad \frac{m_t}{m} \\
\text{if } dD_t > 0 & \quad \text{if } dD_t < 0 \\
- \frac{\partial V}{\partial M_t} & = -v'(m_t) = P_t \\
- \frac{\partial V}{\partial M_t} & = -v'(m_t) = P_t (1 + \chi)
\end{align*}
\]
Optimal Platform Payout and Buy-back (burn) $dD_t$

\[
\begin{align*}
\frac{m}{\bar{m}} \quad & \quad \frac{m_t}{\bar{m}} \\
\text{if } dD_t > 0 \\
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\[
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\[
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Franchise (continuation) value \(\rightarrow\) Resistance against over-supply
Coase (1972): Producers of durable goods are always tempted to meet the residual demand until the product price falls to marginal cost.
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• Consumers wait for the lowest price
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- Producers of durable goods are always tempted to meet the residual demand until the product price falls to marginal cost  
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  - Consumers rationally form expectation of token price
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  - Consumers rationally form expectation of token price
- Producers sell all goods immediately at price equal to $MC$
  - Producers sell $\infty$ tokens immediately at price equal to 0?
Difference: • Token demand is not stationary – $A_t$ grows geometrically, so future demand is stronger – users cannot expect $P_t$ falls to 0
  ▪ Bulow (1982), Stokey (1981)
Difference: • Token demand is *not stationary* – \( A_t \) grows geometrically, so future demand is stronger – users cannot expect \( P_t \) falls to 0
  ▪ Bulow (1982), Stokey (1981)
• Real option concern: \( A_t \) grows stochastically, and increasing token supply can only be reversed costly due to \( \chi \)
Difference: • Token demand is *not stationary* – $A_t$ grows geometrically, so future demand is stronger – users cannot expect $P_t$ falls to 0
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• Real option concern: $A_t$ grows stochastically, and increasing token supply can only be reversed costly due to $\chi$

**Platform resists excess supply**

$$m_t = \frac{M_t}{A_t} \in [\underline{m}, \bar{m}]$$

*Incentive to buyback and burn tokens*
Luxury brands including Burberry burn stock worth millions
Outline

- Introduction
- Model and Solution
- Franchise Value as Discipline
- **Token Overhang**
- The Value of Commitment
- Conclusion
Optimal Platform Investment $L_t$

\[
\frac{\partial V}{\partial A_t} A_t \mu^H + \frac{\partial^2 V}{\partial A_t^2} A_t^2 (\sigma^H)^2 L_t = \frac{\partial F}{\partial L_t} \left( \frac{\partial V/\partial M_t}{P_t} \right)
\]

**Marginal contribution to $V$**

**Marginal cost**

**Marginal cost of investment:**

\[
\frac{\partial F}{\partial L_t}
\]
Optimal Platform Investment $L_t$

$$\frac{\partial V}{\partial A_t} A_t \mu^H + \frac{\partial^2 V}{\partial A_t^2} \sigma^H A_t^2 L_t = \frac{\partial F}{\partial L_t} \left( \frac{\partial V}{\partial M_t} \right)$$

**Marginal contribution to $V$**

**Marginal cost**

*Marginal cost of investment:* $\frac{\partial F}{\partial L_t}$

*Dynamic token issuance cost:* $-\frac{\partial V}{\partial M_t} > 1$, at $\bar{m}$, $-\frac{\partial V}{\partial M_t} = P_t (1 + \chi)$

*Underinvestment!*
Conflict of Interest and Under-investment

Investment paid by new tokens → User convenience ↑
**Conflict of Interest and Under-investment**

Investment paid by new tokens → User convenience ↑ → Can platform seize all surplus via token price ↑?
Conflict of Interest and Under-investment

Investment paid by new tokens → User convenience ↑ → Can platform seize all surplus via token price ↑?  
NO!

User heterogeneity + High \( u_i \) keep surplus; only the marginal user breaks even

One token price integrated market
Conflict of Interest and Under-investment

Investment paid by new tokens \( \rightarrow \) User convenience ↑

User heterogeneity +

One token price integrated market

\[ m_t = \frac{M_t}{A_t} \downarrow \text{if negative shock} \]

Closer to \( \bar{m} \) costly buy-back

Can platform seize all surplus via token price ↑? \( \text{NO!} \)

High \( u_i \) keep surplus; only the marginal user breaks even

Platform pays \( \chi \) and cannot share it with users
Conflicts of Interest and Under-investment

Investment paid by new tokens → User convenience ↑ → Can platform seize all surplus via token price ↑? NO!

User heterogeneity +

One token price integrated market

Closer to $m$ costly buy-back

Platform bears $\chi$ and cannot share it with users

$$m_t = \frac{M_t}{A_t} \downarrow \text{if negative shock}$$
Token Overhang

A: Dynamic Token Issuance Cost

$$\frac{-V_{M_t}}{P_t}$$

B: Platform Investment

$$L_t$$

Token Supply / Platform Productivity $$m_t$$
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**Time Inconsistency**

*A rule of investment set at \( t = 0 \) \( \rightarrow \) higher \( V \) in every state*

\[
\frac{dM_t}{M_t} = \mu^M dt \quad \text{at} \quad m_t \in (\underline{m}, \overline{m}), \text{s.t.,} \quad \tilde{L}(m_t) > L_t
\]

Higher token value dominates the cost of more frequent token burning
**Value Function: Discretion vs. Commitment**

**A: Platform Owner Value – Discretion**

- $v(m_t)$ vs. Token Supply / Platform Productivity $m_t$

**B: Platform Owner Value – Commitment**

- $v(m_t)$ vs. Token Supply / Platform Productivity $m_t$
Time Inconsistency

A rule of investment set at $t = 0 \rightarrow$ higher $V$ in every state

$$\frac{dM_t}{M_t} = \mu^M dt \text{ at } m_t \in (\underline{m}, \bar{m}) \text{, s.t., } \bar{L}(m_t) > L_t$$

Commitment via Blockchain
Conclusion: Token-Based Corporate Finance

- A model of token-based ecosystem
  - Endogenous token supply and platform development
  - Endogenous token price and user-base formation

1. Platform franchise value $\rightarrow$ discipline on token supply ("dilution")
   - Durable-good problem, because of endogenous platform development
     - Token burning contributes to token price stability; stablecoin without collateral-backing (in the paper)

2. Token overhang
   - Ingredients: (a) integrated token market (one price), (b) user heterogeneity,
     (c) stochastic investment outcome, (d) financial friction

3. The value of commitment under token overhang
   - Blockchain enables token as means of payment and financing tools
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Related Papers

- Platforms without tokens: Rochet and Tirole (2003), Stulz (2019)


- Tokens for users and contributors with exogenous supply: Sockin and Xiong (2018), Pagnotta (2018) among others


- Money: (1) convenience yield in Baumol-Tobin models, Krishnamurthy and Vising-Jørgensen (2012); (2) demand with inflation expectation in Cagan (1956); (3) financing tools in Bolton and Huang (2016)
Users and Token Demand

- Price-taking, in equilibrium $dP_t = P_t \mu_t^P dt + P_t \sigma_t^P dZ_t$

- Maximize the NPV, given $r$, the cost of capital

$$\mathbb{E} \left[ \int_{t=0}^{\infty} e^{-rt} dy_{i,t} \right],$$

where

$$dy_{i,t} = \max \left\{ 0, \max_{k_{i,t}>0} \left[ \left( P_t k_{i,t} \right)^{1-\alpha} \left( N_t^\gamma A_t u_i \right)^{\alpha} dt + \text{convenience} \right] + k_{i,t} \mathbb{E}_t \left[ dP_t \right] - \phi dt - P_t k_{i,t} r dt \right\}$$

- Deadweight access cost $\phi dt$: cognitive, application integration etc.
Users and Token Demand

- Price-taking, in equilibrium $dP_t = Pt\mu_t^P dt + Pt\sigma_t^P dZ_A$
- Maximize the NPV, given $r$, the cost of capital

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- Deadweight access cost $\phi dt$: cognitive, application integration etc.
Users and Token Demand (con’t)

- Agent $i$’s optimal holding of tokens is given by

$$k_{i,t}^* = \frac{N_t^\gamma A_t u_i}{P_t} \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \quad (2)$$

It has the following properties:
1. $k_{i,t} \uparrow$ in $N_t$, user base.
2. $k_{i,t} \downarrow$ in token price $P_t$.
3. $k_{i,t} \uparrow$ in $A_t$, platform usefulness, and agent-specific $u_i$.
4. $k_{i,t} \uparrow$ in the expected token price change, $\mu_t^P$.

- Determine $N_t$: if profits $> 0$, agents participate

- Adoption: maximized profit $N_t^\gamma A_t u_i \alpha \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1-\alpha}{\alpha}} > \phi$
  - A threshold value of $u_i$ above which users adopt
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Token Valuation

- Users’ aggregate transaction need: $U_t := \int_{u \geq u_t} u g(u) \, du$, where $u_t$ is the indifference threshold.

- Token market clearing, $M_t = \int_{i \in [0,1]} k_{i,t}^* \, di$.

- The equilibrium token price is given by

$$P_t = \frac{N_t^{\gamma} U_t A_t}{M_t} \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}.$$  \hspace{1cm} (3)

- $\mu_t^P$ is the expectation of risk-adjusted token appreciation.
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Optimal Token Supply

- Two controls: $L_t$ (investment) and $D_t$ (payout/buy-back)
- Two state variables: $M_t$ and $A_t$

\[
V_t = \max_{\{L_t, D_t\}_{s \geq t}} \int_{s=t}^{+\infty} \mathbb{E}_t \left[ e^{-r(s-t)} P_s dD_s \left[ I\{dD_s \geq 0\} - (1 + \chi) I\{dD_s < 0\} \right] \right],
\]

- Continuation value: the present value of seigniorage
# Calibration

### Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Model</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Key Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) $\alpha$</td>
<td>0.3</td>
<td>Comovement: $N_t$ &amp; $P_t$</td>
<td>Cong, Li, and Wang (2018a)</td>
</tr>
<tr>
<td>(2) $\mu^H$</td>
<td>50%</td>
<td>Productivity growth</td>
<td>Cong, Li, and Wang (2018a)</td>
</tr>
<tr>
<td>(3) $\sigma^H$</td>
<td>200%</td>
<td>Productivity volatility</td>
<td>Cong, Li, and Wang (2018a)</td>
</tr>
<tr>
<td>(4) $\theta$</td>
<td>$1e4$</td>
<td>Investment variation</td>
<td>Illustrative purpose</td>
</tr>
<tr>
<td>(5) $\xi$</td>
<td>2</td>
<td>The Distribution of $u_i$</td>
<td>Illustrative purpose</td>
</tr>
<tr>
<td>(6) $\kappa$</td>
<td>0.8</td>
<td>The Distribution of $u_i$</td>
<td>Illustrative purpose</td>
</tr>
<tr>
<td>(7) $\theta$</td>
<td>$5e5$</td>
<td>The Distribution of $u_i$</td>
<td>Comparative Statics – Competition Effects</td>
</tr>
<tr>
<td>(8) $\chi$</td>
<td>20%</td>
<td>Token buyback cost</td>
<td>Comparative Statics – Financial Frictions</td>
</tr>
<tr>
<td>(9) $\gamma$</td>
<td>$1/8$</td>
<td>$N_t$ in total productivity</td>
<td>Parameter restriction</td>
</tr>
<tr>
<td><strong>Panel B: Other Parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10) $r$</td>
<td>5%</td>
<td>Risk-free rate</td>
<td></td>
</tr>
<tr>
<td>(11) $\phi$</td>
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<td>Scaling effect on $A_t$</td>
<td></td>
</tr>
<tr>
<td>(12) $\rho$</td>
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<td>Shock correlation: SDF &amp; $A_t$</td>
<td></td>
</tr>
<tr>
<td>(13) $\eta$</td>
<td>1</td>
<td>Price of risk</td>
<td></td>
</tr>
</tbody>
</table>
Parametric Assumption of $u_i$ Distribution

- $u_i$ follows a Pareto distribution on $[U_t, +\infty)$ with c.d.f.

$$G_t(u) = 1 - \left( \frac{U_t}{u} \right)^\xi,$$

where $\xi \in (1, 1/\gamma)$ and $U_t = 1/(\omega A_t^\kappa)$, $\omega > 0, \kappa \in (0, 1)$.

- The cross-section mean of $u_i$ is $\frac{\xi U_t}{\xi - 1}$

- $U_t$ decreases in $A_t$: (1) to capture competition effects; (2) for analytical convenience
Endogenous User Base

Proposition

Given $\mu_t^P$, we have a unique non-degenerate solution for $N_t$ under the Pareto distribution of $u_i$ given by Equation (4):

$$N_t = \left( \frac{A_t^{1-\kappa \alpha}}{\omega \phi} \right)^{\frac{\xi}{1-\xi \gamma}} \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{\xi}{1-\xi \gamma}} \left( \frac{1-\alpha}{\alpha} \right),$$

if $A_t^{1-\kappa} \left( \frac{1-\alpha}{r - \mu_t^P} \right)^{\frac{1-\alpha}{\alpha}} \leq \frac{\omega \phi}{\alpha}$; otherwise, $N_t = 1$.

- Why hold token? (1) Usage $A_t$. (2) Investment $\mu_t^P$
Optimal Control

HJB equation:

\[ r V (M_t, A_t) \, dt = \max_{L_t, dD_t} V_{M_t} \left[ \frac{F (L_t, A_t)}{P_t} \, dt + dD_t \right] + V_{A_t} A_t L_t \mu^H \, dt \]

\[ + \frac{1}{2} V_{A_t} A_t^2 L_t^2 \sigma^2 \, dt + P_t dD_t \left[ I \{dD_t \geq 0\} - (1 + \chi) I \{dD_t < 0\} \right], \]

with

\[ dM_t = \frac{F (L_t, A_t)}{P_t} \, dt + dD_t, \]

\[ \text{and} \quad \frac{dA_t}{A_t} = \left( \mu^L \, dt + \sigma^L dZ_t \right) L_t \]

Proposition

The optimal token supply is given by (1) the optimal choice of \( L_t \),

\[ L_t^* = \frac{V_{A_t} \mu^H + V_{M_t} \frac{1}{P_t}}{-V_{M_t} \frac{\theta}{P_t} - V_{A_t} A_t \sigma^2}, \tag{6} \]

and (2) the optimal choice of \( dD_t \) — the platform pays out token dividends \( (dD_t^* > 0) \) if \( P_t \geq -V_{M_t} \), and the insiders buy back and burn tokens out of circulation \( (dD_t^* < 0) \) if \( -V_{M_t} \geq P_t (1 + \chi) \).
Risk-Neutral to Physical Measure

- SDF: \( \frac{d\Lambda_t}{\Lambda_t} = -rdt - \eta d\hat{Z}_t^\Lambda \)

- Risk-neutral measure: \( dZ_t^\Lambda = d\hat{Z}_t^\Lambda + \eta dt \).

- \( \rho = corr(dZ_t^\Lambda, dZ_t^A) \)

- Calibrate the model to the speed of \( N_t \) growth in data
  - Drift of \( A_t \) under physical measure: \( \mu^A + \eta \rho \sigma^A \)